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S-branes and (anti-)bubbles in (A)dS space

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Abstract: We describe the construction of new locally asymptotically (A)dS geometries with relevance for the AdS/CFT and dS/CFT correspondences. Our approach is to obtain new solutions by analytically continuing black hole solutions. A basic consideration of the method of continuation indicates that these solutions come in three classes: S-branes, bubbles and anti-bubbles. A generalization to spinning or twisted solutions can yield spacetimes with complicated horizon structures. Interestingly enough, several of these spacetimes are nonsingular.

Keywords: AdS-CFT and dS-CFT Correspondence, Black Holes in String Theory, Black Holes.

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1. Introduction

Time-dependent backgrounds in string theory provide an interesting arena for exploring intrinsically dynamical issues such as black hole evaporation, cosmological evolution or the possible formation and resolution of singularities. An essential ingredient in understanding quantum gravity in asymptotically Anti-de Sitter (AdS) spacetimes is the Maldacena conjecture (or the AdS/Conformal Field Theory (CFT) correspondence) [1−3].[1] In this framework, a *large* black hole in AdS is described as a thermal state of the dual conformal

[1] It is referred to as a duality in the sense that the supergravity (closed string) description of D-branes and the field theory (open string) description are different formulations of the same physics.

field theory. A remarkable property of the AdS/CFT correspondence is that it works even far from the conformal regime [4, 5]. This result is consistent with the interpretation of the radial coordinate of AdS space as a energy scale of the dual CFT. In other words, timelike D-branes lead to a spacelike holography.

Inspired by the fact that the microphysical statistical origin of cosmological horizon entropy may well be associated with a holographic dual theory, some authors conjectured a de Sitter/CFT correspondence [6 − 8] — the bulk time translation is dual to the boundary scale transformation and so the time is holographically reconstructed. Using the analogy with D-branes, one expects new (spacelike) objects S-branes to be at the heart of the dS/CFT correspondence. An S-brane [9] is a topological defect for which time is a transverse dimension and so it exists only for a moment (or brief period) of time. In the same way that (for $\Lambda = 0$) p-branes are stationary solutions of supergravity and string theory, S-branes are time-dependent backgrounds of the theory.

In this paper we find three families of exact solutions: S-branes, bubble-like solutions and the newly coined anti-bubble solutions. Roughly speaking, in D dimensions, these solutions involved a $(D-2)$-dimensional hyperbolic space, de Sitter, or anti-de Sitter component, respectively. The solutions are classified according to the technique of their construction. (See also [10].)

The first is the S-brane type [11 − 18] describing a shell of radiation coming in from infinity and creating an unstable brane which subsequently decays.[2] Nonspinning S-branes solutions involve $\mathbf{H}_{D-2}$ which can be quotiented, to yield topological (A)dS black holes — these have been known (see e.g. [20, 21] and references therein). For example, a spherical black hole with $\Lambda > 0$, under analytic continuation and sign flip of metric, gives a black solution with $\Lambda < 0$ and a hyperbolic component. We may refer to this as a (topological) black hole in AdS, or as the corresponding S-brane solution to a black hole in dS. On the other hand, a BHAdS with $\Lambda < 0$ yields a cosmologically singular S-BHAdS with $\Lambda > 0$; this solution has an exterior region [22] which is time-dependent, like de Sitter space itself.[3] The S-BH(A)dS solutions have timelike singularities and the Penrose diagrams are related to a $\pi/2$-rotation of the corresponding black hole Penrose diagrams [20]. However, in the Reissner-Nordstrøm case, the inmost horizon is moved to negative r and the $r > 0$ S-brane Penrose diagram has fewer regions. The solutions we describe here are analogs of the S-branes found previously (with $\Lambda = 0$) and that justifies the terminology.

With $\Lambda = 0$, a black hole is stationary and an S-brane is time-dependent, but a $\Lambda \neq 0$ will dominate at large r and its sign determines the signature of the Killing vector. Black holes and S-branes in AdS are both stationary, and black holes and S-branes in dS are both time-dependent.

[2]The solutions in [12] do not have horizons and are better described as gravitational wave solutions, describing the creation and decay of a fluxbrane. They were constructed by analytic continuation keeping in mind Sen's rolling tachyon solution for unstable D-branes [19].

[3]We emphasize a solution being time-dependent if it does not have an exterior/asymptotic region with a timelike Killing vector. Thus the Schwarzschild black hole is not time-dependent even though there is no global timelike Killing vector, and de Sitter space is time-dependent.

The second family are of bubble type [23–27]. A bubble is a $(D-3)$-sphere which exists only for $r \geq r_{\min}$. An x^D Killing circle vanishes at $r = r_{\min}$. These bubbles are time-dependent since the $(D-3)$-sphere expands in a de Sitter fashion. We also define 'double bubbles' as solutions where an expanding $(D-3)$-sphere exists over an interval $r_{\min} \leq r \leq r_{\max}$ and the x^D-circle closes at both endpoints (hence two 'bubbles of nothing').

The third family is the newly coined anti-bubbles, which must be distinguished from expanding bubbles. Here, we have AdS_{D-2} whose spatial section is not a sphere but a noncompact 'anti-bubble.' This spatial section exists only for $r \geq r_{\min}$. We also find double anti-bubbles where the AdS_{D-2} runs over $r_{\min} \leq r \leq r_{\max}$.

When rotation parameters are added (hence looking at Kerr-(A)dS solutions), there is an additional complication to the solution where a quantity W (or Δ_θ) can vanish. This can generate additional horizons changing the time-dependent or stationary nature of various regions in the solution. Also, sometimes this will close the spacetime creating boundary conditions with inconsistent Killing compactifications.

In some cases involving rotation, there are two types of S-branes. For example, the $D = 4$ Kerr solution admits a usual S-brane [13] and also a $\pi/2$-S-brane [18]. This is analogous to the double Killing Kerr bubble (with horizons and CTCs) as opposed to the $\theta \to \pi/2 + i\theta$ Kerr bubble (without horizons or CTCs). The idea is that in even dimensions, one direction cosine is not associated with a rotation and hence it is different; in any dimension, direction cosines with rotations turned off are different from those with rotations turned on. We will be careful to emphasize when such different solutions are available.

The rest of the paper is organized as follows: In section 2 we look at the simple case of RN(A)dS black holes in $D = 4$ and find the bubbles, S-branes and anti-bubbles (as well as interior double bubbles and anti-bubbles) using card diagram[4] techniques and using $r\theta$ diagrams. Then in section 3 we look at the general-D RN(A)dS solutions, finding bubbles, S-branes and anti-bubbles. We see how the conformal boundary geometry of the S-brane fits nicely with that for the bubble to give the global boundary of AdS.

In section 4 we move to the Kerr solutions. These solutions are sometimes plagued by what we call $W = 0$ coordinate singularities [28] (also called $\Delta_\theta = 0$ singularities [26]). We find that these are just spinning Killing horizons (or twisted closures of spatial Killing circles), which complicate the structure of the spacetime. We allow general rotation parameters and try to avoid $W = 0$ singularities. Then, in section 5, we only turn one rotation on and allow $W = 0$ singularities. Here we find extremely interesting global structures for bubble geometries with $W = 0$ singularities and illustrate them by drawing skeleton diagrams[5] for the θ coordinate.

[4]Card diagrams are applicable for $D = 4$ or 5 black hole spacetimes which have the requisite 2 or 3 Killing fields. Card diagrams and the technique of the γ-flip were used to understand S-branes in [17].

[5]A skeleton diagram is a 1-dimensional analog of a card diagram; it shows only the coordinate which determines where the horizons are. The skeleton diagram for the Schwarzschild black hole is a $+$, where the horizontal legs are where r is spacelike and the vertical legs where r is timelike. Four legs meet at a nonextremal horizon.

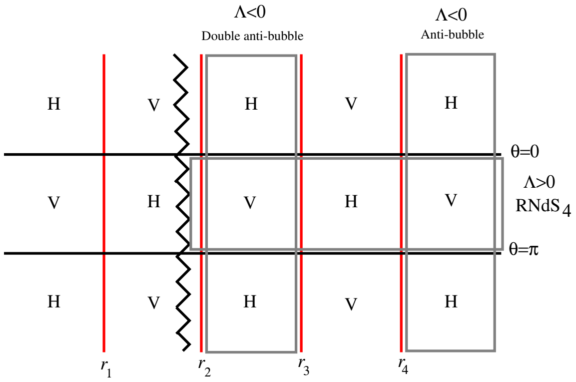

Figure 1: Those regions of the extended RNdS_4 spacetime, where we do not send $Q \to iQ$. We have the black hole with $\Lambda > 0$, the anti-bubble with $\Lambda < 0$, and the $r_2 \leq r \leq r_3$ double anti-bubble with $\Lambda < 0$. This $r\theta$ diagram is similar to the C-metric diagrams in [30].

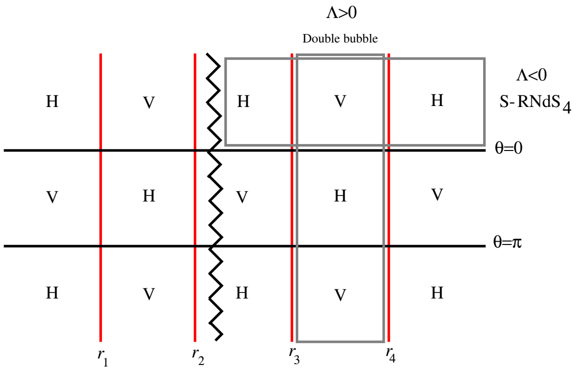

Figure 2: Those regions of the extended RNdS_4 spacetime, where we send $Q \to iQ$. We have the $r_3 \leq r \leq r_4$ double bubble with $\Lambda > 0$, and the $\mathrm{S\text{-}RNdS}_4$ with $\Lambda < 0$.

We conclude in section 6 by outlining the role of these solutions in the holographic AdS/CFT correspondence (or the putative dS/CFT correspondence). Lastly we give a short appendix on generalized card diagrams as they apply to pure $(\mathrm{A})\mathrm{dS}_D$ space for $D = 4, 5$.

We will not look at genus-zero planar or toroidal black holes or their generalizations — see [20, 21] and references contained therein.

2. 4d examples

Before writing down the general analytic continuation, we look at the simple case of four dimensions. Here (as well as in five dimensions) black holes in (A)dS are of Weyl type: in D dimensions they have $D-2$ commuting Killing fields. Methods of obtaining bubbles, anti-bubbles and S-branes are then very evident (see figures 1–4). Unlike previous approaches to analytic continuation to S-branes (involving continuations like $r \to it$), we will find all the spacetimes by only performing simple analytic continuations involving real sections of

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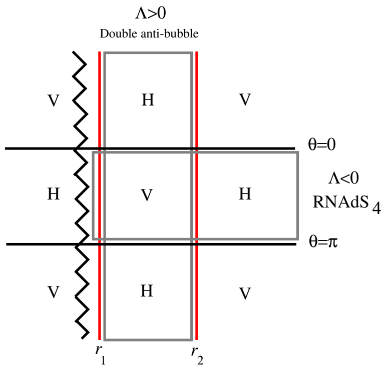

Figure 3: Those regions of the extended RNAdS$_4$ spacetime, where we do not send $Q \to iQ$.

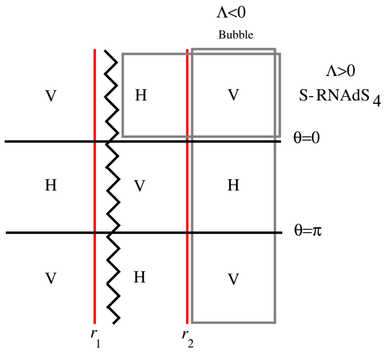

Figure 4: Those regions of the extended RNAdS$_4$ spacetime, where we send $Q \to iQ$.

hyperboloids, by making sign flips in the metric and sometimes continuing to imaginary charge.

We begin discussing the variety of solutions we will obtain starting with the 4d Reissner-Nordstrøm-(A)dS black hole solution. From the 4d RNdS black hole with $\Lambda > 0$ we can obtain an S-brane with $\Lambda < 0$ as well as a static 'anti-bubble' (which is to an expanding bubble what AdS$_2$ is to dS$_2$) with $\Lambda < 0$. From the 4d RNAdS black hole $\Lambda < 0$ we can obtain a bubble solution with $\Lambda < 0$ as well as an S-brane with $\Lambda > 0$.

2.1 De Sitter

The RNdS$_4$ solution with $\Lambda = (D-1)/l^2$, $\Lambda > 0$ is

$$\begin{aligned} ds^2 &= -f(r)(dx^4)^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2\,, \\ A &= Q\frac{dx^4}{r}\,, \end{aligned} \tag{2.1}$$

where $f(r) = 1 - 2M/r + Q^2/r^2 - r^2/l^2$. We take the horizon function to be the quartic polynomial $r^2 f(r) = -r^4/l^2 + r^2 - 2Mr + Q^2$, which can have up to four roots[6] $r_1 < 0 < r_2 \le r_3 \le r_4$. The root r_4 is the cosmological horizon of de Sitter, and r_3 and r_2

[6] See [20] for a discussion of roots and parameters. However, the triple root $r_2 = r_3 = r_4$ is singular.

are the outer and inner black hole horizons, which can coincide in an 'extremal' case. The singularity is at $r = 0$, and $r < 0$ with its single (cosmological) horizon r_1 represents a negative-mass black hole in dS_4. We can draw the two non-Killing directions r, θ on the diagram in figure 1. The $RNdS_4$ black hole solution occupies the middle row, to the right of the singularity. On this and also figure 2 we anticipate several solutions that can be obtained from $RNdS_4$ by trivial analytic continuation.

We can explicitly give $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$. An alternative way to write it (to make contact with later formulas) is to set $-1 \leq \mu_0 = \cos\theta \leq 1$ and $0 \leq \mu_1 = \sin\theta \leq 1$; we have the constraint $\mu_0^2 + \mu_1^2 = 1$.

This 4d spacetime has two commuting Killing vectors (say, the x^4 and ϕ directions) and so is of generalized Weyl type. The regions in figure 1 are labelled H(orizontal) and V(ertical) in analogy with Weyl card diagrams [17] (and see the appendix to this paper). Horizontal cards are stationary regions of spacetimes, whereas vertical cards are time-dependent and have $D-2$ commuting spatial Killing fields. There are two basic operations one can perform on cards. On a horizontal card, one can do a double Killing continuation to pick a new time direction. On vertical cards, one can perform an operation known as a γ-flip which exchanges the timelike and spacelike character of the two non-Killing directions. The γ-flip can be realized by changing the sign of the metric $g_{\mu\nu} \to -g_{\mu\nu}$ and analytically continuing all Killing directions; here $x^4 \to ix^4$ and $\phi \to i\phi$. Along with the sign flip of the metric, from the Einstein-Maxwell-Λ equation we must flip the sign of Λ. Our conventions are to leave the parameter l alone but now interpret the solution to solve a $\Lambda < 0$ equation. The sign flip of the metric also forces the gauge field strength to become imaginary; but we also continue the 1-form $dx^4 \to idx^4$, so the net result is a real field strength. In summary, the γ-flip takes a signature $(D-1,1)$ vertical card with a real field strength and given Λ and turns it 'on its side' to yield a signature $(D-1,1)$ vertical card with a real field strength and opposite sign of Λ.

It is clear then that we can take the vertical-card $r \geq r_4$ region and turn the card on its side with a γ-flip. We now occupy the right column of figure 1. The solution is

$$\begin{aligned} ds^2 &= -f(r)(dx^4)^2 - \frac{dr^2}{f(r)} - r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \\ &\to -f(r)(dx^4)^2 - \frac{dr^2}{f(r)} + r^2 d\theta^2 - r^2 \sinh^2\theta d\phi^2 \,. \end{aligned}$$

We decompactify ϕ to get horizons at $\theta = 0, \pi$; the $0 \leq \theta \leq \pi$ variable can be continued $\theta \to i\theta$, $\theta \to \pi + i\theta$ to give a patched representation of AdS_2. We must compactify $x^4 \simeq x^4 + 4\pi|f'(r_4)|^{-1}$ to avoid a conical singularity at $r = r_4$. In summary, we have AdS_2 and an x^4-circle fibered over $r \geq r_4$. At $r = r_4$ the x^4-circle closes in a fashion very similar to well-known expanding bubble solutions giving a minimum-size AdS_2. In analogy with 'bubble' terminology we shall call this solution [25] the $RNdS_4$ anti-bubble, with $\Lambda < 0$.

Note that we could also perform the γ-flip on $r_2 \leq r \leq r_3$. Now the spacetime has two boundaries $r = r_2, r_3$ where the x^4-circle closes. We must then match $f'(r_2) + f'(r_3) = 0$ to eliminate conical singularities at both ends; then of the parameters M, Q and l, one is dependent — one can also be eliminated by a global conformal transformation, leaving one true dimensionless shape parameter. This solution occupies the center column in figure 1, and we call it the $r_2 \leq r \leq r_3$ RNdS$_4$ double anti-bubble. Since it does not have an $r \to \infty$ asymptotic region, it is not useful for holography.

The region $r_3 \leq r \leq r_4$ is a stationary region (horizontal card) and we may perform a double Killing continuation $x^4 \to ix^4$, $\phi \to i\phi$ to get a new solution. We must also continue $Q \to iQ$ to make the field strength real. Then the horizon function $r^2 f(r) = 1 - 2M/r - Q^2/r^2 + r^2/l^2$ is changed and its roots are generically $r_1 \leq r_2 < 0 < r_3 \leq r_4$. We now reference solutions to figure 2; note that positive- and negative-mass solutions are qualitatively similar. The solution is

$$\begin{aligned} ds^2 &= f(r)(dx^4)^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 - \sin^2\theta\, d\phi^2) \\ &\to f(r)(dx^4)^2 + \frac{dr^2}{f(r)} + r^2(-d\theta^2 + \sinh^2\theta\, d\phi^2)\,. \end{aligned} \tag{2.2}$$

We see now a patched dS$_2$; and the x^4 circle vanishes at $r = r_3, r_4$, so we require $f'(r_3) + f'(r_4) = 0$ and have the $r_3 \leq r \leq r_4$ RNdS$_4$ double bubble. It solves $\Lambda > 0$ Einstein-Maxwell-Λ. Since it does not have an $r \to \infty$ asymptotic region, it is not useful for holography.

We can however take the vertical card at hyperbolic θ on the RNdS$_4$ double bubble and perform a γ-flip. The resulting solution has $\Lambda < 0$ and we call it S-RNdS$_4$, since it is the S-brane gotten from the RNdS$_4$ geometry. It occupies the top row of figure 2, to the right of the singularity. It is

$$ds^2 = f(r)(dx^4)^2 - \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sinh^2\theta d\phi^2)\,.$$

We see $\mathbf{H}_2$, azimuthally parametrized. Note that just as the RNdS$_4$ black hole is not stationary, its S-brane is not time-dependent.

The RNdS$_4$, RNdS$_4$ anti-bubble and S-RNdS$_4$ all have $r \to \infty$ asymptotic regions where they are locally asymptotic to (A)dS$_4$, depending on their sign of Λ.

We summarize the five spacetimes gotten from RNdS$_4$ in the following table.

Name	Λ	Hyp.	iQ?	ϕ cpct	x^4 cpct	Asym.
RNdS$_4$	$+$	S^2	No	Yes	No	dS$_4$
RNdS$_4$ anti-bub.	$-$	AdS$_2$	No	No	Yes	AdS$_4$
RNdS$_4$ doub. bub.	$+$	dS$_2$	Yes	No	Double	
RNdS$_4$ doub. anti-bub.	$-$	AdS$_2$	No	No	Double	
S-RNdS$_4$	$-$	$\mathbf{H}_2$	Yes	Yes	No	AdS$_4$

Here we give the name, the sign of the cosmological constant, the real section of the complex 2-hyperboloid, whether Q has been continued with $Q \to iQ$, whether ϕ is compact, whether x^4 is compact, whether x^4 has two boundaries instead of just one since this is a nontrivial condition and whether the manifold asymptotes locally to (A)dS$_4$. The isometry group of the solution is the isometry of the hyperbolic space (SO(3), SO$(2,1)$ or SO$(1,2)$) times $\mathbb{R}$ (if x^4 is noncompact) or $U(1)$ (if x^4 is compact).

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2.2 Anti-de Sitter

To achieve RNAdS_4, take $l \to il$ in (2.1). We have

$$\begin{aligned} ds^2 &= -f(r)(dx^4)^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2\,, \\ A &= Q\frac{dx^4}{r}\,, \end{aligned} \tag{2.3}$$

where $f(r) = 1 - 2M/r + Q^2/r^2 + r^2/l^2$. Then $r^2 f(r)$ can have at most two roots; we assume $Q \neq 0$ and M is large enough so that this happens. Then $0 < r_1 \leq r_2$ — see figure 3. Looking ahead, when we will have to send $Q \to iQ$, we will have $r_1 < 0 < r_2$ — see figure 4.

Since our discovery of solutions parallels the RNdS_4 case, we will be brief. We can take $r \geq r_2$ which is static, and double Killing rotate $x^4 \to ix^4$, $\phi \to i\phi$, $Q \to iQ$. We then get the RNAdS_4 bubble solution with a patched description of dS_2 fibered over $r \geq r_2$, and an x^4-circle which closes at $r = r_2$ to give the bubble. This is the right column of figure 4.

We can perform a γ-flip on the upper (vertical card) region of the RNAdS_4 bubble to achieve S-RNAdS_4, which is the top row of figure 4, to the right of the singularity. It has azimuthally parametrized $\mathbf{H}_2$ and $\Lambda < 0$.

Finally we can perform a γ-flip on $r_1 \leq r \leq r_2$ of the RNAdS_4 black hole to achieve a double anti-bubble. It has AdS_2, $\Lambda > 0$, and appears as the central column in figure 3.

The RNAdS_4 black hole, bubble and S-brane asymptote to $\mathrm{(A)dS}_4$ locally.

Again we summarize

Name	Λ	Hyp.	iQ?	ϕ cmpct	x^4 cmpct	Asym.
RNAdS_4	$-$	S^2	No	Yes	No	AdS_4
RNAdS_4 bub.	$-$	dS_2	Yes	No	Yes	AdS_4
RNAdS_4 doub. anti-bub.	$+$	AdS_2	No	No	Double	
S-RNAdS_4	$+$	$\mathbf{H}_2$	Yes	Yes	No	dS_4

The fact that all the listed solutions are different from each other is evident just by looking at where they stand in relation to their neighbors and the singularity, in figures 1–4. One can also use the symmetry groups to prove they are different.

These 4d solutions with SO(3) symmetry can also yield bubbles, anti-bubbles or S-branes based on the continuation $\theta \to \pi/2 + i\theta$ instead of $\theta \to i\theta$. These solutions are not different from those solutions just described. However, even-dimensional Kerr solutions admit $\theta \to \pi/2 + i\theta$ solutions which are different from those gotten by taking the analog of $\theta \to i\theta$. This distinction has been emphasized in [17, 18] and we will revisit it below.

The card-diagram method of the γ-flip also applies in 5d. Card diagrams of $\mathrm{(A)dS}_4$ and $\mathrm{(A)dS}_5$ spacetime can be drawn; in fact due to their extra symmetries, many different card diagram representations exist. These diagrams are also useful for visualizing the local-$\mathrm{(A)dS}_4$, AdS_5 asymptotia of the black hole, bubble, anti-bubble, and S-brane solutions. For some details and diagrams, see appendix to this paper.

However, card diagram methods do not apply in $D \geq 6$. Nonetheless, analogs of these RN solutions do exist in all $D \geq 4$, and we give them in the next section.

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3. General Reissner-Nordstrøm-(A)dS$_D$ solutions

We construct the S-branes for the general RN(A)dS$_D$ solution, along with the bubbles and anti-bubbles. We will not have figures 1–4 to guide us, but again we will only do simple analytic continuations involving cos(h)-type quantities, metric sign flips, and $Q \to iQ$. We will not focus on double (anti-)bubble solutions, only on those solutions with $r \to \infty$ asymptotia. We also give the $r \to \infty$ conformal boundary geometry (CBG) gotten from the given coordinates.

3.1 De Sitter

The RNdS$_D$ black hole is

$$\begin{aligned} ds^2 &= -f(r)(dx^D)^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{D-2}\,, \\ A &= \sqrt{\frac{D-2}{2(D-3)}}\frac{Q dx^4}{r}\,, \end{aligned} \tag{3.1}$$

where $f(r) = 1 - 2M/r^{D-3} + Q^2/r^{2(D-3)} - r^2/l^2$, $\Lambda = (D-1)/l^2 > 0$. The horizon function is the polynomial $r^{2(D-3)}f(r)$. For D even, an appropriate parameter subdomain gives roots $r_1 < 0 < r_2 \leq r_3 \leq r_4$ — we consider the solution for $r > 0$. For D odd we let r^2 be the independent variable and allow it to go negative. Then for the appropriate parameter subdomain the horizon function has roots which we call $0 < r_2^2 \leq r_3^2 \leq r_4^2$ and we consider $r^2 > 0$ (there is no r_1^2 root in our notation).

Take $D = 2n+2$ even first; we can write

$$d\Omega^2_{D-2} = d\mu_0^2 + d\mu_1^2 + \cdots + d\mu_n^2 + \mu_1^2 d\phi_1^2 + \cdots + \mu_n^2 d\phi_n^2\,, \tag{3.2}$$

where $-1 \leq \mu_0 \leq 1$ and $0 \leq \mu_i \leq 1$ $i = 1, \ldots, n$. The constraint is $\mu_0^2 + \sum_{i=1}^n \mu_i^2 = 1$.

To get the S-brane, we send $\mu_i \to i\mu_i$, $i = 1, \ldots, n$, send $g_{\mu\nu} \to -g_{\mu\nu}$, and $Q \to iQ$. The line element (3.2) becomes, including the sign flip, the (still spacelike)

$$d\mathbf{H}^2_{D-2} = -d\mu_0^2 + d\mu_1^2 + \cdots + d\mu_n^2 + \mu_1^2 d\phi_1^2 + \cdots + \mu_n^2 d\phi_n^2 \tag{3.3}$$

with constraint $\mu_0^2 - \sum_{i=1}^n \mu_i^2 = 1$. The solution is

$$\begin{aligned} ds^2 &= f(r)(dx^D)^2 - \frac{dr^2}{f(r)} + r^2 d\mathbf{H}^2_{D-2}\,, \\ A &= \sqrt{\frac{D-2}{2(D-3)}}\frac{Q dx^4}{r}\,, \end{aligned}$$

which has $\Lambda < 0$. As $r \to \infty$, the solution is asymptotically locally AdS$_D$. The conformal boundary geometry (CBG) in these coordinates is $ds^2 = -(dx^D)^2/l^2 + d\mathbf{H}^2_{D-2}$, which is $\mathbb{R}_{\text{time}} \times \mathbf{H}_{D-2}$. There is no invariant relating the size of these two components. The horizon function now has roots $r_1 \leq r_2 < 0 < r_3 \leq r_4$ like the S-RNdS$_4$ case. Our $r \to \infty$ gets the asymptotia gotten from one Rindler wedge of the r_4 horizon — there is another Rindler wedge ignored in this procedure.

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Taking the S-brane, we can take $x^D \to ix^D$, continue back $Q \to -iQ$, so $r_1 < 0 < r_2 \leq r_3 \leq r_4$. We compactify $x^D \simeq x^D + 4\pi|f'(r_4)|^{-1}$ to form an anti-bubble at $r = r_4$, and then take for example $\phi_1 \to i\phi_1$. Thus

$$ds^2 = -f(r)(dx^D)^2 - \frac{dr^2}{f(r)} + r^2 d\mathrm{AdS}^2_{D-2}.$$

Here,

$$d\mathrm{AdS}^2_{D-2} = -d\mu_0^2 + d\mu_1^2 + \cdots + d\mu_n^2 - \mu_1^2 d\phi_1^2 + \mu_2^2 d\phi_2^2 + \cdots + \mu_n^2 d\phi_n^2$$

is a patched description; we can go through the μ_1 Rindler horizon to $\mu_1 \to i\mu_1$. This anti-bubble solution was discovered in planar coordinates in [31] where they were termed fluxbranes. In planar coordinates the solutions resemble branes but keeping in mind their global structure, we choose not to think of them as branes, the term anti-bubble (for the AdS_{D-2} factor) being more appropriate.

The CBG is $ds^2 \propto (dx^D)^2/l^2 + d\mathrm{AdS}^2_{D-2}$, which is $S^1 \times \mathrm{AdS}_{D-2}$. Since x^D is compact, there is a dimensionless invariant, the ratio of the circumference of the x^D-circle to the unit-sized AdS_{D-2}.

In odd dimension $D = 2n+1$ the idea is the same, but the cosines are set up differently. We have μ_i, $i = 1, \ldots, n$, with $0 \leq \mu_i \leq 1$ and

$$d\Omega^2_{D-2} = d\mu_1^2 + \cdots + d\mu_n^2 + \mu_1^2 d\phi_1^2 + \cdots + \mu_n^2 d\phi_n^2 \,. \tag{3.4}$$

To get the S-brane, take $\mu_i \to i\mu_i$ $i = 1, \ldots, n-1$, $g_{\mu\nu} \to -g_{\mu\nu}$, $Q \to iQ$, flip the sign of Λ, and $\phi_n \to i\phi_n$. Then we have

$$d\mathbf{H}^2_{D-2} = d\mu_1^2 + \cdots + d\mu_{n-1}^2 - d\mu_n^2 + \mu_1^2 d\phi_1^2 + \cdots + \mu_n^2 d\phi_n^2 \,, \tag{3.5}$$

and the geometry is

$$ds^2 = f(r)(dx^D)^2 - \frac{dr^2}{f(r)} + r^2 d\mathbf{H}^2_{D-2}.$$

The horizon function now has roots $r_2^2 < 0 < r_3^2 \leq r_4^2$.

To go to the anti-bubble, send $x^D \to ix^D$, return $Q \to -iQ$, and send say $\phi_1 \to i\phi_1$. Then

$$d\mathrm{AdS}^2_{D-2} = d\mu_1^2 + \cdots + d\mu_{n-1}^2 - d\mu_n^2 - \mu_1^2 d\phi_1^2 + \mu_2^2 d\phi_2^2 + \cdots + \mu_n^2 d\phi_n^2 \,.$$

This is in the 'real' μ_1 patch where the constraint reads $\mu_n^2 - \mu_1^2 - \mu_2^2 - \cdots - \mu_{n-1}^2 = 1$, but going through the Rindler horizon at $\mu_1 = 0$, we send $\mu_1 \to i\mu_1$ and get $\mu_n^2 + \mu_1^2 - \mu_2^2 - \cdots - \mu_{n-1}^2 = 1$. The anti-bubble is

$$ds^2 = -f(r)(dx^D)^2 - \frac{dr^2}{f(r)} + r^2 d\mathrm{AdS}^2_{D-2} \,.$$

The CBG is $ds^2 \propto (dx^D)^2/l^2 + d\mathrm{AdS}^2_{D-2}$, which is $S^1 \times \mathrm{AdS}_{D-2}$. Since x^D was compactified at the largest r-root, there is a dimensionless invariant, the circumference of the x^D-circle over the unit AdS size.

3.2 Anti-de Sitter

From the RNAdS$_D$ solution we will make a $\Lambda < 0$ bubble and $\Lambda > 0$ S-brane. The solution is like for RNdS$_D$ except $f(r) = 1-2M/r^{D-3}+Q^2/r^{2(D-3)}+r^2/l^2$, with roots $0 < r_1 \leq r_2$.

For $D = 2n+2$, $d\Omega^2_{D-2}$ is as in (3.2). To get the bubble, send $x^D \to ix^D$, compactify at r_2, and send say $\phi_1 \to i\phi_1$. The solution is

$$ds^2 = f(r)(dx^D)^2 + \frac{dr^2}{f(r)} + d\mathrm{dS}^2_{D-2}\,, \tag{3.6}$$

where

$$d\mathrm{dS}^2_{D-2} = d\mu_0^2 + d\mu_1^2 + \cdots + d\mu_n^2 - \mu_1^2 d\phi_1^2 + \mu_2^2 d\phi_2^2 + \cdots + \mu_n^2 d\phi_n^2\,.$$

Of course we can go through the Rindler horizon at $\mu_1 = 0$ and send $\mu_1 \to i\mu_1$. This bubble was described in [25, 26].

The CBG is $ds^2 \propto (dx^D)^2/l^2 + d\mathrm{dS}^2_{D-2}$ which is $S^1\times\mathrm{dS}_{D-2}$. There is a dimensionless invariant, the ratio of the sizes of these factors.

To get the S-brane from the black hole, send $\mu_i \to i\mu_i$, $i = 1,\ldots,n$, $g_{\mu\nu} \to -g_{\mu\nu}$, $Q \to iQ$, and flip the sign of Λ. Now $r_1 < 0 < r_2$; the solution is

$$ds^2 = f(r)(dx^D)^2 - \frac{dr^2}{f(r)} + r^2 d\mathbf{H}^2_{D-2}\,, \tag{3.7}$$

where

$$d\mathbf{H}^2_{D-2} = -d\mu_0^2 + d\mu_1^2 + \cdots + d\mu_n^2 + \mu_1^2 d\phi_1^2 + \cdots + d\mu_n^2\,.$$

The constraint is $\mu_0^2 - \sum_{i=1}^n \mu_i^2 = 1$. Since the singularity is just protected by a cosmological horizon r_2, this solution is nakedly singular, like the S-Schwarzschild geometry of pure Einstein theory.

The CBG is $ds^2 \propto (dx^D)^2/l^2 + d\mathbf{H}^2_{D-2}$, which is $\mathbb{R} \times \mathbf{H}_{D-2}$. This is a Euclidean geometry, so this would serve to investigate the putative dS/CFT correspondence.

In odd dimensions $D = 2n+1$, we have $d\Omega^2_{D-2}$ given by (3.4). The bubble is gotten by sending $x^D \to ix^D$, $Q \to iQ$ and say $\phi_1 \to i\phi_1$. The solution is (3.6), where

$$d\mathrm{dS}^2_{D-2} = d\mu_1^2 + \cdots + d\mu_n^2 - \mu_1^2 d\phi_1^2 + \mu_2^2 d\phi_2^2 + \cdots + \mu_n^2 d\phi_n^2\,.$$

The odd-dimensional S-brane on the other hand is gotten from $\mu_i \to i\mu_i$ $i = 1,\ldots,n-1$, $g_{\mu\nu} \to -g_{\mu\nu}$, $\phi_n \to i\phi_n$. The solution is (3.7) with the hyperbolic space given by (3.5).

3.3 Extremal S-RNdS$_D$

Take S-RNdS$_D$,

$$\begin{aligned} ds^2 &= f(r)(dx^D)^2 - \frac{dr^2}{f(r)} + r^2 d\mathbf{H}^2_{D-2} \\ A &= \sqrt{\frac{D-2}{2(D-3)}}\frac{Q dx^D}{r}\,. \end{aligned} \tag{3.8}$$

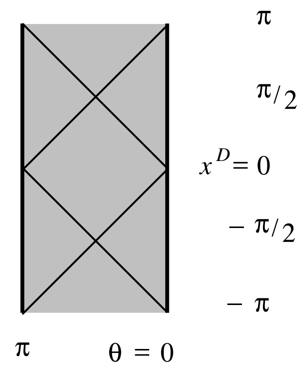

Figure 5: Penrose diagram for the global conformal boundary of AdS_D. The top and bottom $x^D = \pm\pi$ are identified. Each diamond is filled in by the asymptotia of an RNAdS_D bubble, and $\mathrm{S\text{-}RNdS}_D$ gives triangles. Four triangles and two diamonds neatly fit together so their closure gives the whole global boundary.

Here, $f(r) = 1 - 2M/r^{D-3} - Q^2/r^{2(D-3)} - r^2/l^2$. For D even we normally assume four roots $r_1 \le r_2 < 0 < r_3 \le r_4$. For D odd we have $r_2^2 < 0 < r_3^2 \le r_4^2$. In either case, one can find an extremal solution where $r_3 = r_4$. Here, 'extremal' refers just to degenerate horizons; this solution is the analog of the $r_3 = r_4$ maximal RNdS_D black hole solution where the outer black hole horizon coincides with the cosmological (de Sitter) horizon. Then $f(r) \sim -A(r-r_4)^2$, and letting $\epsilon = r - r_4$, we can take a scaling limit where $\epsilon \to 0$, ϵx^D fixed, which is

$$ds^2 = -A\epsilon^2 (dx^D)^2 + \frac{d\epsilon^2}{A\epsilon^2} + r_4^2 d\mathbf{H}_{D-2}^2 \,,$$

and $F \propto \frac{Q}{r_4^{D-2}} dx^D \wedge d\epsilon$. This solution is $\mathrm{AdS}_2 \times \mathbf{H}_{D-2}$. Thus extremal $\mathrm{S\text{-}RNdS}_D$ interpolates between $\mathrm{AdS}_2 \times \mathbf{H}_{D-2}$ at the extremal horizon to local AdS_D at $r = \infty$ and the latter can have a CBG of $\mathbf{H}_{D-2} \times \mathbb{R}_{\mathrm{time}}$.

Solutions which interpolate between spacetimes with similar $\mathbf{H}_{D-2}$ factors were found in [32]. For de Sitter bounces, see [33].

3.4 Embedding the conformal boundary geometry of bubbles and S-branes

In [26] the conformal boundary of the RNAdS_D bubble was given as a subset of $S^1_{\mathrm{time}} \times S^{D-2}$ which is the global conformal boundary of AdS_D — we have identified the time-circle in the canonical fashion. There, it was found that in the $x^D\theta$ strip, where $0 \le \theta \le \pi$ is the polar angle for the S^{D-2} of RNAdS_D, the bubble asymptotes to the open set $|\theta - \pi/2| < |x^D - \pi/2|$; each bubble asymptotes to one diamond in figure 5. We now complete the picture by showing that $\mathrm{S\text{-}RNdS}_D$ asymptotes to the remainder triangles.

First, take $D = 2n+1$ odd. The embedding of dS_D into $\mathbb{R}^{D,1}$ is

$$\begin{aligned} X'^0 &= \sqrt{1-r^2}\sinh x^D \\ X^1 &= r\mu_1 \cos\phi_1 \\ X^2 &= r\mu_1 \sin\phi_1 \end{aligned}$$

$$
\begin{aligned}
&\vdots\\
X^D &= \pm\sqrt{1-r^2}\cosh x^D\,.
\end{aligned}
$$

Here, a prime denotes a timelike coordinate. We want to send $\mu_i \to i\mu_i$ for $i = 1,\dots,n-1$, $\phi_n \to i\phi_n$, and flip the sign of $g_{\mu\nu}$. Then we get, upon also taking $r > 1$,

$$
\begin{aligned}
X'^0 &= \sqrt{r^2-1}\sinh x^D\\
X^1 &= r\mu_1\cos\phi_1\\
X^2 &= r\mu_1\sin\phi_1\\
&\vdots\\
X'^{D-2} &= r\mu_n\cosh\phi_n\\
X^{D-1} &= r\mu_n\sinh\phi_n\\
X^D &= \pm\sqrt{r^2-1}\cosh x^D\,.
\end{aligned}
$$

Then $X'^{D-2} > 0$ and we let $X'^0/X'^{D-2} = T$. The global-time angle $\tan^{-1}T$ runs from $-\pi/2$ to $\pi/2$. Then $\tan|\theta-\pi/2| = |X^D|/\sqrt{(X^{D-1})^2+(X^1)^2+(X^2)^2+\cdots} = (\cosh x^d)/(\mu_n\cosh\phi_n) > |T|$, so we have precisely two triangles from this description.

For $D = 2n+2$ even, we have

$$
\begin{aligned}
X'^0 &= \sqrt{1-r^2}\sinh x^D\\
X^1 &= r\mu_1\cos\phi_1\\
X^2 &= r\mu_1\sin\phi_1\\
&\vdots\\
X^{D-1} &= r\mu_0\\
X^D &= \pm\sqrt{1-r^2}\cosh x^D\,.
\end{aligned}
$$

Sending $\mu_i \to i\mu_i$ for $i = 1,\dots,n$ and flipping the sign of $g_{\mu\nu}$, and going to $r > 1$, we get

$$
\begin{aligned}
X'^0 &= \sqrt{r^2-1}\sinh x^D\\
X^1 &= r\mu_1\cos\phi_1\\
X^2 &= r\mu_1\sin\phi_1\\
&\vdots\\
X'^{D-1} &= r\mu_0\\
X^D &= \pm\sqrt{r^2-1}\cosh x^D\,.
\end{aligned}
$$

We have $X'^{D-1} > 0$ so set $X'^0/X'^{D-1} = (\sinh x^D)/\mu_0 = T$, and $-\infty < T < \infty$. Furthermore, $|X^D|/\sqrt{(X^1)^2+(X^2)^2+\cdots} = (\cosh x^D)/\mu_0 > |T|$.

4. Kerr-(A)dS$_D$ and related solutions which avoid $W = 0$

We now find S-branes, bubbles and anti-bubbles from the Kerr-(A)dS solutions. In the notation of [28], for bubbles, S-branes and anti-bubbles, a quantity W defined below has the possibility to zero along certain hypersurfaces which are r-independent — they depend on the cosines and hyperbolic functions.

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In this section, we will look for solutions which avoid $W = 0$, which are clearly good solutions, with an expected global structure. The $W = 0$ coordinate singularity for the Kerr-AdS$_4$ bubble was a source of some confusion in [26]; actually it is a regular spinning horizon with a constant angular velocity. Following their approach, in section 5 we will look at the case of general D with one angular momentum turned on, and explore solutions where we allow $W = 0$. In this case there are two nontrivial Killing directions and one nontrivial cosine, like the $D = 4$ case. A treatment of general D, general a_i will not be given here.

4.1 Black holes, S-branes: odd dimensions

In odd dimensions $D = 2n+1$, there are n angular momentum parameters a_i, $i = 1, \ldots, n$ for a spinning black hole, and we want to turn off just one of them, $a_n = 0$. This will force a horizon/polar-origin/orbifold for both the black hole and S-brane at $r^2 = 0$.

For $\lambda = 1/l^2$ for $\Lambda = (D-1)\lambda > 0$, the Kerr-dS solution of [28] is

$$\begin{aligned} ds^2 =& -W(1-\lambda r^2)(dx^D)^2 + \frac{U dr^2}{V-2M} + \frac{2M}{U}\left(dx^D - \sum_{i=1}^{n-1}\frac{a_i\mu_i^2 d\phi_i}{1+\lambda a_i^2}\right)^2 + \\ &+ \sum_{i=1}^{n}\frac{r^2+a_i^2}{1+\lambda a_i^2}\left(d\mu_i^2 + \mu_i^2(d\phi_i - \lambda a_i dx^D)^2\right) + \\ &+ \frac{\lambda}{W(1-\lambda r^2)}\left(\sum_{i=1}^{n}\frac{(r^2+a_i^2)\mu_i d\mu_i}{1+\lambda a_i^2}\right)^2, \end{aligned} \tag{4.1}$$

where $V = \frac{1}{r^2}(1-\lambda r^2)\prod_{i=1}^n(r^2+a_i^2)$, $W = \sum_{i=1}^n \frac{\mu_i^2}{1+\lambda a_i^2}$ and $U = \sum_{i=1}^n \frac{\mu_i^2}{r^2+a_i^2}\prod_{j=1}^n(r^2+a_j^2)$. The constraint is $\sum_{i=1}^n \mu_i^2 = 1$ and for reference, $F = U/V = \frac{r^2}{1-\lambda r^2}\sum_{i=1}^n \frac{\mu_i^2}{r^2+a_i^2}$. The thermodynamics of these black holes are discussed in [29].

To get the S-brane, continue $\mu_i \to i\mu_i$ for $i = 1, \ldots, n-1$, $\phi_n \to i\phi_n$ and perform a flip $g_{\mu\nu} \to -g_{\mu\nu}$. The change in the sign of the metric necessitates a change in the cosmological constant — our notation will be $\Lambda < 0$ but $1/l^2 = \lambda > 0$. So the S-Kerr-dS$_D$ solves Einstein's equations with $\Lambda < 0$. The constraint is now $\mu_n^2 - \sum_{i=1}^{n-1}\mu_i^2 = 1$. The solution is

$$\begin{aligned} ds^2 =& \, W(1-\lambda r^2)(dx^D)^2 - \frac{U dr^2}{V-2M} - \frac{2M}{U}\left(dx^D + \sum_{i=1}^{n-1}\frac{a_i\mu_i^2 d\phi_i}{1+\lambda a_i^2}\right)^2 + \\ &+ \sum_{i=1}^{n-1}\frac{r^2+a_i^2}{1+\lambda a_i^2}\left(d\mu_i^2 + \mu_i^2(d\phi_i - \lambda a_i dx^D)^2\right) - r^2(d\mu_n^2 - \mu_n^2 d\phi_n^2) - \\ &- \frac{\lambda}{W(1-\lambda r^2)}\left(r^2\mu_n d\mu_n - \sum_{i=1}^{n-1}\frac{(r^2+a_i^2)\mu_i d\mu_i}{1+\lambda a_i^2}\right)^2. \end{aligned} \tag{4.2}$$

Now, W and U are given in terms of μ_i by

$$W = \mu_n^2 - \sum_{i=1}^{n-1}\frac{\mu_i^2}{1+\lambda a_i^2}$$

and

$$U = \left(\mu_n^2 - \sum_{i=1}^{n-1} \frac{\mu_i^2}{1 + a_i^2/r^2}\right) \prod_{j=1}^{n-1} (r^2 + a_j^2)\,.$$

Note that all elements in the solution that are functions of r, a_i and λ have not changed and so it is clear that the S-brane has the same parameter regions and ensuing horizon structure as the black hole. However, the description of each r-interval as stationary or time-dependent is flipped. In particular, horizons are still located at $V - 2M = 0$. The determinant of the $(n+1)\times(n+1)$ Killing submetric has $\det g_{\alpha\beta} = r^2(V-2M)W \prod_{i=1}^n \frac{\mu_i^2}{1+\lambda a_i^2}$, which has opposite sign from the black hole case [28].

Consider the horizon function

$$r^2(V - 2M) = r^2 \left((1 - \lambda r^2) \prod_{i=1}^{n-1} (r^2 + a_i^2) - 2M\right).$$

The geometry depends only on r^2, which we take to be the independent variable, and allow to be negative. In the ordinary parameter range $2M > \prod_{i=1}^{n-1} a_i^2 = A^2$ then for sufficiently small λ, there are two positive roots $0 < r_1^2 \le r_2^2$. For $r^2 > r_2^2$, the solution is stationary as we expect for asymptotically AdS_D space. With $r_1^2 < r^2 < r_2^2$ the solution is time-dependent and for $r_1^2 = r_2^2$, there is an extremal horizon with an AdS_2 scaling limit. For $0 < r^2 < r_1^2$ the solution is stationary. At $r^2 = 0$, the solution is not singular and the ϕ_n-circle closes with periodicity[7]

$$\phi_n \simeq \phi_n + 2\pi \sqrt{\frac{A^2}{(A^2 - 2M)}}\,.$$

Since $r^2 \ge 0$, there is no $U = 0$ singularity.

For the anomalous parameter range $0 < 2M < A^2$ (described for $\lambda = 0$ in [18] and in great detail for $D = 5$ black holes and S-branes), any $\lambda > 0$ is allowed and we have a root $r^2 = r_2^2 > 0$, a ϕ_n Milne horizon at $r^2 = 0$ and another root at $r^2 = -r_1^2 < 0$. Without loss of generality we assume $0 < a_1 \le a_2 \le \cdots \le a_{n-1}$ and so $-a_1^2 < -r_1^2 < 0$. The spacetime closes at $r^2 = -r_1^2$ — the twisting and periodicity can be gotten by continuing the angular momentum and surface gravity of [28]. In particular, just put $r_{\text{horizon}} \to ir_1$ ($r_{\text{horizon}} \to -ir_1$ also gives the right answer) and $\kappa \to \pm i\kappa$. We have

$$\Omega^i = \frac{a_i(1 + \lambda a_i^2)}{a_i^2 - r_1^2}\,, \qquad \kappa = r_1(1 + \lambda r_1^2) \sum_{i=1}^{n} \frac{1}{a_i^2 - r_1^2} + \frac{1}{r_1}\,.$$

For $n = 2$, $\lambda = 0$, this reduces to $\Omega = a/2M$, $\kappa = \frac{\sqrt{a^2-2M}}{2M}$, which matches [18]. Since $-r^2 < a_1^2 \le a_2^2 \le \cdots$, there is no $U = 0$ singularity.

The region $-r_1^2 \le r^2 \le 0$ for an anomalous range S-Kerr-dS gives upon $\phi_n \to i\phi_n$ the new S-Kerr instanton of [18]. The extremal case $2M = A^2$ is nonsingular at $r^2 = r_1^2 = 0$ and has a dS_3 scaling limit as described in [18].

[7] The global structure here for D odd is just like the $\lambda = 0$ case, which was first discovered by Jones and Wang [18].

For the Kerr-dS black hole, $\lambda > 0$ and $W = \sum_{i=1}^{n} \frac{\mu_i^2}{1+\lambda a_i^2}$ never zeroes. For λ a little negative, namely $-\min_i(a_i^{-2}) < \lambda < 0$, W is still positive. This is the Kerr-AdS black hole which avoids $W = 0$. For $\lambda < -\min_i(a_i^{-2})$, there is a mixture of positive and negative terms and we will find a $W = 0$ coordinate singularity (moreover, a priori the spacetime has the wrong signature).

For S-Kerr-dS, we have $W = \mu_n^2 - \sum_{i=1}^{n-1} \frac{\mu_i^2}{1+\lambda a_i^2}$. For $\lambda > 0$, from the constraint $\mu_n^2 - \sum_{i=1}^{n-1} \mu_i^2 = 1$, this never zeroes; we have a good S-Kerr-dS with $\Lambda < 0$. However, any $\lambda < 0$ will give us a $W = 0$ coordinate singularity.

4.2 Black holes and S-branes: even dimensions

The even-dimensional case $D = 2n + 2$ is quite different from the odd-dimensional case. Here we have n rotation parameters and we want to leave them all on, so the black hole has an equatorial 'ring' singularity and the S-brane is nonsingular at $r = 0$.[8] The black hole solution is

$$
\begin{aligned}
ds^2 = & -W(1-\lambda r^2)(dx^D)^2 + \frac{U dr^2}{V - 2M} + \frac{2M}{U}\left(dx^D - \sum_{i=1}^{n} \frac{a_i \mu_i^2 d\phi_i}{1+\lambda a_i^2}\right)^2 + \\
& + d\mu_0^2 + \sum_{i=1}^{n} \frac{r^2 + a_i^2}{1+\lambda a_i^2} d\mu_i^2 + \sum_{i=1}^{n} \frac{r^2 + a_i^2}{1+\lambda a_i^2} \mu_i^2 (d\phi_i - \lambda a_i dx^D)^2 + \\
& + \frac{\lambda}{W(1-\lambda r^2)} \left(r^2 \mu_0 d\mu_0 + \sum_{i=1}^{n} \frac{(r^2 + a_i^2)\mu_i d\mu_i}{1+\lambda a_i^2}\right)^2 .
\end{aligned}
$$

The constraint is $\mu_0^2 + \sum_{i=1}^{n} \mu_i^2 = 1$, where $-1 \le \mu_0 \le 1$ has no rotation parameter and $0 \le \mu_i \le 1$ for $i = 1, \dots, n$ has rotation parameter ϕ_i. We have $V = \frac{1}{r}(1 - \lambda r^2) \prod_{i=1}^{n} (r^2 + a_i^2)$, $W = \mu_0^2 + \sum_{i=1}^{n} \frac{\mu_i^2}{1+\lambda a_i^2}$, $U = \frac{1}{r}(\mu_0^2 + \sum_{i=1}^{n} \frac{\mu_i^2}{1+a_i^2/r^2}) \prod_{i=1}^{n} (r^2 + a_i^2)$, and for reference $F = U/V = \frac{r^2}{1-\lambda r^2}(\mu_0^2 + \sum_{i=1}^{n} \frac{\mu_i^2}{r^2+a_i^2})$.

For the even-dimensional case, the solution depends properly on r, not r^2. It has the symmetry $r \to -r$, $M \to -M$. For the right range of parameters, the horizon function $rV(r) - 2Mr$ has four roots, $r_1 < 0 < r_2 \le r_3 \le r_4$; an extremal black hole occurs for $r_2 = r_3$. Since all a_i are turned on, at $r = 0$ there is only a $U = 0$ S^{D-3} 'ring' singularity at $\mu_0 = 0$ — hence we can go to negative r.

The continuation to S-brane is $\mu_i \to \mu_i$, $i = 1, \dots, n$, and $g_{\mu\nu} \to -g_{\mu\nu}$. We then have Λ with sign opposite to λ.

$$
\begin{aligned}
ds^2 = & \, W(1-\lambda r^2)(dx^D)^2 - \frac{U dr^2}{V - 2M} - \frac{2M}{U}\left(dx^D + \sum_{i=1}^{n} \frac{a_i \mu_i^2 d\phi_i}{1+\lambda a_i^2}\right)^2 - \\
& - d\mu_0^2 + \sum_{i=1}^{n} \frac{r^2 + a_i^2}{1+\lambda a_i^2} d\mu_i^2 + \sum_{i=1}^{n} \frac{r^2 + a_i^2}{1+\lambda a_i^2} \mu_i^2 (d\phi_i - \lambda a_i dx^D)^2 -
\end{aligned}
$$

[8] The global structure of this solution for the $\lambda = 0$ case was worked out by Lü and Vázquez-Poritz [15]. The $D = 4$ S-Kerr-dS with $\Lambda < 0$ is in [20].

$$-\frac{\lambda}{W(1-\lambda r^2)}\left(r^2\mu_0 d\mu_0 - \sum_{i=1}^{n}\frac{(r^2+a_i^2)\mu_i d\mu_i}{1+\lambda a_i^2}\right)^2 .$$

The quantity V is as for the black hole, $W = \mu_0^2 - \sum_{i=1}^n \frac{\mu_i^2}{1+\lambda a_i^2}$ and $U = \frac{1}{r}(\mu_0^2 - \sum_{i=1}^n \frac{\mu_i^2}{1+a_i^2/r^2}) \times \prod_{i=1}^n (r^2+a_i^2)$. The constraint is now $\mu_0^2 - \sum_{i=1}^n \mu_i^2 = 1$ and this implies $\mu_0 \geq 1$. Since there is no ring singularity, we may follow $r \to -\infty$ and so the solution is nonsingular.

S-Kerr-dS with $\lambda > 0$ avoids $W = 0$. S-Kerr-AdS with $\lambda < 0$ hits a $W = 0$ coordinate singularity.

The $r_2 = r_3$ extremal case of the S-brane is nonsingular at the extremal horizon and has a dS_2 scaling limit [15]. The $r_3 = r_4$ extremal case has an AdS_2 scaling limit.

4.3 Asymptotics

Take say the odd D case. Sending $r \to \infty$ for the black hole, we get a CBG

$$ds^2 \propto \lambda W (dx^D)^2 + \sum_{i=1}^{n}\frac{1}{1+\lambda a_i^2}\left(d\mu_i^2 + \mu_i^2(d\phi_i - \lambda a_i dx^D)^2\right) - \\ -\frac{1}{W}\left(\sum_{i=1}^{n}\frac{\mu_i d\mu_i}{1+\lambda a_i^2}\right)^2 .$$

This appears to be spinning, but the spinning is a pure coordinate effect. If we let $\phi_i = \tilde{\phi}_i + \lambda a_i x^D$, we get

$$ds^2 \propto \lambda W (dx^D)^2 + \sum_{i=1}^{n}\frac{1}{1+\lambda a_i^2}\left(d\mu_i^2 + \mu_i^2 d\tilde{\phi}_i{}^2\right) - \frac{1}{W}\left(\sum_{i=1}^{n}\frac{\mu_i d\mu_i}{1+\lambda a_i^2}\right)^2 .$$

This is the same CBG as we get from the $M = 0$ case, which was identically $(\mathrm{A})\mathrm{dS}_D$. In fact these are just the 'spheroidal coordinates' of [28]. This boundary is conformal to $\mathbb{R}_{\text{space}} \times S^{D-2}$ for $\lambda > 0$ and $\mathbb{R}_{\text{time}} \times S^{D-2}$ for $\lambda < 0$.

The S-brane we know has no $W = 0$ locus for $\lambda > 0$ $(\Lambda < 0)$ and has CBG

$$ds^2 \propto \lambda W (dx^D)^2 + \sum_{i=1}^{n-1}\frac{1}{1+\lambda a_i^2}\left(d\mu_i^2 + \mu_i^2(d\phi_i - \lambda a_i dx^D)^2\right) - \\ - d\mu_n^2 + \mu_n^2 d\phi_n^2 + \frac{1}{W}\left(\mu_n d\mu_n - \sum_{i=1}^{n-1}\frac{\mu_i d\mu_i}{1+\lambda a_i^2}\right)^2 .$$

Again sending $r \to \infty$ has dropped out the M parameter, so this CBG should be conformal to $\mathbb{R}_{\text{time}} \times \mathbf{H}_{D-2}$.

4.4 The μ_0-negative S-Kerr-dS for even dimensions

There is another S-brane obtainable from Kerr-dS_D for D even. This is the analog of the 4d 'Kerr $\pi/2$-bubble on its side' of [18]. A sphere in even D is written as

$$d\Omega_{D-2}^2 = d\mu_0^2 + d\mu_1^2 + \cdots + d\mu_n^2 + \mu_1^2 d\phi_1^2 + \cdots + \mu_n^2 d\phi_n^2 ,$$

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where the constraint is $\mu_0^2 + \sum_{i=1}^n \mu_i^2 = 1$. Send $\mu_0 \to i\mu_0$ and $\mu_i \to i\mu_i$ $i = 2, \ldots, n$, $\phi_1 \to i\phi_1$, and flip the sign of the metric. We then get

$$d\mathbf{H}_{D-2}^2 = d\mu_0^2 - d\mu_1^2 + \cdots + d\mu_n^2 + \mu_1^2 d\phi_1^2 + \cdots + \mu_n^2 d\phi_n^2 \,,$$

where $-\mu_0^2 + \mu_1^2 - \mu_2^2 - \cdots - \mu_n^2 = 1$. In the Kerr case along with $\phi_1 \to i\phi_1$ we must do $a_1 \to ia_1$. We call the resulting solution the μ_1-positive S-Kerr-dS, or a μ_0-negative S-Kerr-dS. This emphasizes that in the $\mathbf{H}_{D-2}$ constraint, it is not μ_0 but rather a μ_i that has a rotation angle, that has the plus sign. The full Kerr S-brane is

$$\begin{aligned} ds^2 = {} & W(1-\lambda r^2)(dx^D)^2 - \frac{U dr^2}{V - 2M} - \frac{2M}{U}\left(dx^D + \frac{a_1\mu_1^2 d\phi_1}{1-\lambda a_1^2} + \sum_{i=2}^n \frac{a_i\mu_i^2 d\phi_i}{1+\lambda a_i^2}\right)^2 + \\ & + d\mu_0^2 - \frac{r^2 - a_1^2}{1-\lambda a_1^2} d\mu_1^2 + \sum_{i=2}^n \frac{r^2 + a_i^2}{1+\lambda a_i^2} d\mu_i^2 + \\ & + \frac{r^2 - a_1^2}{1-\lambda a_1^2}\mu_1^2 (d\phi_1 - \lambda a_1 dx^D)^2 + \sum_{i=2}^n \frac{r^2 + a_1^2}{1+\lambda a_i^2}\mu_i^2 (d\phi_i - \lambda a_i dx^D)^2 - \\ & - \frac{\lambda}{W(1-\lambda r^2)}\left(-r^2\mu_0 d\mu_0 + \frac{r^2 - a_1^2}{1-\lambda a_1^2}\mu_1 d\mu_1 - \sum_{i=2}^n \frac{r^2 + a_i^2}{1+\lambda a_i^2}\mu_i d\mu_i\right)^2 . \end{aligned}$$

Here, $W = -\mu_0^2 + \frac{\mu_1^2}{1-\lambda a_1^2} - \sum_{i=2}^n \frac{\mu_i^2}{1+\lambda a_i^2}$. For $0 < \lambda < 1/a_1^2$, this does not zero, and we never go to imaginary μ_1. There is no $U = 0$ singularity at $r = a_1$, because $\mu_1 \geq 1$. Horizons are given by $r_1 \leq r_2 < 0 < r_3 \leq r_4$. This solution is also important for constructing even-dimensional anti-bubbles.

4.5 Spinning Λ bubble or anti-bubble solutions

It was noticed in [26] that Kerr-AdS bubbles are 'problematic' from the $W = 0$ singularity, even in the simple 4d case. We now check that $W = 0$ always occurs in Kerr-AdS bubbles and find ways to avoid $W = 0$ for anti-bubbles constructed from Kerr-dS.

Bubbles with large-r asymptotia can only come from $\lambda < 0$ ($\Lambda < 0$) black holes, where the large-r region is stationary. Then in even $D = 2n + 2$,

$$W = \mu_0^2 + \sum_{i=1}^n \frac{\mu_i^2}{1+\lambda a_i^2} \,.$$

One way to get a bubble is to send $\mu_0 \to i\mu_0$, $x^D \to ix^D$, and $a_i \to ia_i$ $i = 1, \ldots, n$. This is the analog of the Kerr $\pi/2$-bubble in 4d gotten from $\theta \to \pi/2 + i\theta$, which has no Killing horizons or CTCs. Then

$$W = -\mu_0^2 + \sum_{i=1}^n \frac{\mu_i^2}{1-\lambda a_i^2} \,, \qquad \text{where} \qquad -\mu_0^2 + \sum_{i=1}^n \mu_i^2 = 1 \,.$$

We see that the terms with + signs in W are divided to make them smaller; the $-\mu_0^2$ can dominate and make $W = 0$.

Another type of bubble[9] is gotten by picking one angle from $\phi_1, \ldots, \phi_n$, for example ϕ_1. Take $x^D \to ix^D$, $\phi_1 \to i\phi_1$, $a_i \to ia_i$ $i = 2, \ldots, n$. This is the analog of the $\mathcal{K}_+$ bubble of [17, 18] in 4d obtained from double Killing continuation, which has Killing horizons and CTCs. In our present case, we can continue past $\mu_1 = 0$ to $\mu_1 \to i\mu_1$ and get

$$W = \mu_0^2 - \frac{\mu_1^2}{1+\lambda a_1^2} + \sum_{i=2}^{n} \frac{\mu_i^2}{1-\lambda a_i^2}\,, \qquad \text{where} \qquad \mu_0^2 - \mu_1^2 + \sum_{i=1}^{n} \mu_i^2 = 1\,.$$

Even if $a_1 = 0$, if some a_i is turned on, $W = 0$ still occurs.

For odd D, there is no μ_0. One can check that in any case except no angular momentum, $W = 0$ occurs.[10]

For anti-bubbles, the situation is better — we find Kerr-dS$_D$ anti-bubbles that avoid $W = 0$ for all $D \geq 4$, and an extra one in $D = 4$. The idea is to make the term with the $+$ coefficient in W to have a denominator smaller than the denominator of all $-$ terms. Recall that to take an anti-bubble we start with Kerr-dS$_D$ with $\lambda > 0$. Take D even and first go to the usual S-brane with $\Lambda < 0$:

$$W = \mu_0^2 - \sum_{i=1}^{n} \frac{\mu_i^2}{1+\lambda a_i^2}\,, \qquad \text{where} \qquad \mu_0^2 - \sum_{i=1}^{n} \mu_i^2 = 1\,.$$

Then pick say the angle ϕ_1. Send $x^D \to ix^D$, $\phi_1 \to i\phi_1$, $a_i \to ia_i$ $i = 2, \ldots, n$. We have

$$W = \mu_0^2 - \frac{\mu_1^2}{1+\lambda a_1^2} - \sum_{i=2}^{n} \frac{\mu_i^2}{1-\lambda a_i^2}\,, \qquad \text{where} \qquad \mu_0^2 - \sum_{i=1}^{n} \mu_i^2 = 1\,.$$

This hits $W = 0$ unless we turn off a_i (with $i = 2, \ldots, n$), but in that case going through the horizon to $\mu_1 \to i\mu_1$,

$$W = \mu_0^2 + \frac{\mu_1^2}{1+\lambda a_1^2} - \sum_{i=2}^{n} \mu_i^2\,.$$

This still hits zero unless $D = 4$ where $i = 2, \ldots, n$ don't exist. So the $D = 4$ case with a turned on, works. This solution can be easily obtained by card diagram methods using figure 1 by performing a γ-flip on the $r \geq r_4$, $0 \leq \theta \leq \pi$ region (and extending to $\theta \to i\theta$ or $\theta \to \pi + i\theta$).

In even D, we can also find a whole family of anti-bubbles from μ_0-negative S-branes. Picking μ_1 to have the $+$ sign, this S-brane has

$$W = -\mu_0^2 + \frac{\mu_1^2}{1-\lambda a_1^2} - \sum_{i=2}^{n} \frac{\mu_i^2}{1+\lambda a_i^2}\,, \qquad \text{where} \qquad -\mu_0^2 + \mu_1^2 - \sum_{i=2}^{n} \mu_i^2 = 1\,.$$

[9]These two types of bubbles are not the same as the two solutions presented at the beginning of [34], in the context of one angular momentum on. There, the first is a bubble with $W = 0$, and the second is an anti-bubble with $W = 0$; its dS$_{D-5}$ is part of an AdS$_{D-4}$ which is part of a perturbed AdS$_{D-2}$. The construction of an anti-bubble by continuing from hyperbolic space suggests those authors also considered the S-brane.

[10]In [26], (31)-(34) should be corrected to have $\Delta_\tau = 1 - \frac{\alpha^2}{l^2}\sinh^2\tau - \frac{\beta^2}{l^2}\cosh^2\tau$, so $W \propto \Delta_\tau = 0$ also occurs.

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Then double Killing $x^D \to ix^D$, $\phi_1 \to i\phi_1$, $a_i \to ia_i$ (with $i = 2, \ldots, n$), we have

$$W = -\mu_0^2 + \frac{\mu_1^2}{1-\lambda a_1^2} - \sum_{i=2}^{n} \frac{\mu_i^2}{1-\lambda a_i^2}\,, \qquad \text{where} \qquad -\mu_0^2 + \mu_1^2 - \sum_{i=2}^{n} \mu_i^2 = 1\,.$$

For $0 < 1-\lambda a_1^2 < 1-\lambda a_i^2$ $i = 2, \ldots, n$, we avoid $W = 0$. Note that we never get to a $\mu_1 = 0$ ϕ_1-horizon, so the above distribution of hyperbolic pieces is global. So for general even D we have this Kerr-dS anti-bubble. We stress that the $D = 4$ solution of this is different from the one gotten from the ordinary S-brane. This present $D = 4$ solution can be obtained from the black hole by $\theta \to \pi/2 + i\theta$, performing a γ-flip to make the non-Killing directions $++$, and then $\phi \to i\phi$, $a \to ia$. To avoid the $U = 0$ 'ring' singularity, taking $a_1 > a_2 \geq \cdots$, we want the largest horizon root (the anti-bubble) to occur at $r_4 > a_2$. This can always be arranged for large enough l.

In odd dimension $D = 2n+1$, the S-brane with a_n turned back on has

$$W = \frac{\mu_n^2}{1-\lambda a_n^2} - \sum_{i=1}^{n-1} \frac{\mu_i^2}{1+\lambda a_i^2}\,, \qquad \text{where} \qquad \mu_n^2 - \sum_{i=1}^{n-1} \mu_i^2 = 1\,.$$

To get a good anti-bubble, we send $x^D \to ix^D$, $\phi_n \to i\phi_n$, $a_i \to ia_i$ $i = 1, \ldots, n-1$, hence

$$W = \frac{\mu_n^2}{1-\lambda a_n^2} - \sum_{i=1}^{n-1} \frac{\mu_i^2}{1-\lambda a_i^2}\,.$$

For $0 < 1-\lambda a_n^2 < 1-\lambda a_i^2$, we avoid $W = 0$. Again, we never reach a $\mu_n = 0$ ϕ_n-horizon so the above characterization is global. The $D = 5$ solution can be obtained by γ-flipping the $r \geq r_4$, $0 \leq \theta \leq \pi/2$ of the black hole, going to $\mu_1 \to i\mu_1$ where $\mu_1 = \sin\theta$, then continuing $\phi_i \to i\phi_i$, $a_i \to ia_i$ (with $i = 1, 2$). Taking $a_n^2 > a_1^2 \geq \cdots$, the largest root (anti-bubble) is at $r_4^2 > a_1^2$ for large l, so the solution is nonsingular.

5. Kerr-(A)dS$_D$: one a_i on, and allow $W = 0$

When a $W = 0$ coordinate singularity occurs, an extra horizon-like locus will be present. We will just look at the case of one angular momentum on, where the Kerr-(A)dS$_D$ solution simplifies considerably [35]. For bubbles, we find that in $D \geq 4$ there is one family of nonsingular solutions and in $D = 4$ there is an additional solution. For S-Kerr-AdS, $D \geq 4$ we find one family. There are no other generically nonsingular solutions.

5.1 Bubbles

Let's examine bubbles; take $D = 4$ first. The Kerr-AdS$_4$ black hole solution [36] is

$$\begin{aligned} ds^2 = {}& \rho^2 \left(\frac{dr^2}{\Delta} + \frac{d\theta^2}{1-(a^2/l^2)\cos^2\theta} \right) - \frac{\Delta}{\rho^2} \left(dx^4 - \frac{a\sin^2\theta}{1-a^2/l^2} d\phi \right)^2 + \\ & + \frac{\sin^2\theta(1-(a^2/l^2)\cos^2\theta)}{\rho^2} \left(a dx^4 - \frac{r^2+a^2}{1-a^2/l^2} d\phi \right)^2 , \end{aligned} \tag{5.1}$$

where $\Delta = (r^2+a^2)(1+r^2/l^2) - 2Mr$, $\rho^2 = r^2 + a^2\cos^2\theta$, and $0 \leq \theta \leq \pi$.

The $\pi/2$-bubble is gotten from $\theta \to \pi/2 + i\theta$, $a \to ia$, $x^4 \to ix^4$. Then

$$ds^2 = \rho^2\left(\frac{dr^2}{\Delta} - \frac{d\theta^2}{1-(a^2/l^2)\sinh^2\theta}\right) + \frac{\Delta}{\rho^2}\left(dx^4 - \frac{a\cosh^2\theta}{1+a^2/l^2}d\phi\right)^2 + \\ + \frac{\cosh^2\theta(1-(a^2/l^2)\sinh^2\theta)}{\rho^2}\left(adx^4 + \frac{r^2-a^2}{1+a^2/l^2}d\phi\right)^2, \tag{5.2}$$

where $\Delta = (r^2-a^2)(1+r^2/l^2) - 2Mr$ has roots $r_1 < 0 < r_2$, and $\rho^2 = r^2 + a^2\sinh^2\theta$. At $r = r_2$, $\Delta = 0$ and the differential displacement $adx^4 + \frac{r_2^2-a^2}{1+a^2/l^2}d\phi = 0$ is null. So let $\tilde{\phi} = \phi - \Omega x^4$, $\tilde{x}^4 = x^4$, $\Omega = -a(1+a^2/l^2)/(r_2^2-a^2)$. Then $d\tilde{\phi} = 0$ is null so the vector $(\partial/\partial\tilde{x}^4)_{\tilde{\phi}}$ is null at $r = r_2$. We can compactify $\tilde{x}^4 \simeq \tilde{x}^4 + \beta$ for some periodicity to make $r = r_2$ the origin of polar coordinates. We must leave $\tilde{\phi}$ noncompact to get a horizon at $W = 0$ (and make no reference to the previous $x^4\phi$ Killing coordinates).[11] The metric is now

$$ds^2 = \rho^2\left(\frac{dr^2}{\Delta} - \frac{d\theta^2}{1-(a^2/l^2)\sinh^2\theta}\right) + \frac{\Delta}{\rho^2}\left(dx^4 - \frac{a\cosh^2\theta}{1+a^2/l^2}(d\tilde{\phi}+\Omega d\tilde{x}^4)\right)^2 + \\ + \frac{\cosh^2\theta(1-(a^2/l^2)\sinh^2\theta)}{\rho^2}\left(ad\tilde{x}^4 + \frac{r^2-a^2}{1+a^2/l^2}(d\tilde{\phi}+\Omega d\tilde{x}^4)\right)^2. \tag{5.3}$$

Setting $\sinh\theta_0 = l/|a|$, this metric has expected bubble properties for $-\theta_0 < \theta < \theta_0$. Following [26], set $\sinh\theta = l/a - \epsilon^2$; then for small real ϵ, we have

$$ds^2 \approx (r^2+l^2)\frac{dr^2}{\Delta} - \frac{2la(l^2+r^2)}{l^2+a^2}d\epsilon^2 + \frac{\Delta}{r^2+l^2}\left(d\tilde{x}^4 - \frac{l^2}{a}(d\tilde{\phi}+\Omega d\tilde{x}^4)\right)^2 + \\ + \epsilon^2\frac{2(l^2+a^2)}{la(l^2+r^2)}\left(ad\tilde{x}^4 + \frac{r^2-a^2}{1+a^2/l^2}(d\tilde{\phi}+\Omega d\tilde{x}^4)\right)^2. \tag{5.4}$$

At $\epsilon = 0$, $d\tilde{x}^4 - \frac{l^2}{a}(d\tilde{\phi}+\Omega d\tilde{x}^4) = 0$ is null. That is a regular spinning horizon — the angular velocity of a regular spinning horizon must be constant [22, 28]. On this side of the horizon the Killing submetric has signature $++$ and on the other side it will have $+-$. We let $\tilde{\tilde{x}}^4 = \tilde{x}^4 - \mho\tilde{\phi}$, $\tilde{\tilde{\phi}} = \tilde{\phi}$, where $\mho = \Omega^{-1}(1-a^2/l^2)$. Then $d\tilde{\tilde{x}}^4 = 0$ is null, or the vector $(\partial/\partial\tilde{\tilde{\phi}})_{\tilde{\tilde{x}}^4}$ gives us the Milne trajectories.

This horizon is then just like a Kerr horizon except the role of the two Killing metric squares is reversed and r is replaced by θ. If we repress the Killing directions and $r \geq r_2$, we arrive at a spacetime skeleton diagram for just the θ coordinate — see figure 6. The vertical legs have θ timelike and the horizontal legs have θ spacelike. The spacetime is periodic in time; each dS_{D-2}-type region gives way to horizons beyond which are stationary regions. The $r \to \infty$ limit gives a CBG which can be represented by the same skeleton diagram — the metric is

$$ds^2 \propto \frac{1}{l^2}\left(d\tilde{x}^4 - \frac{a\cosh^2\theta}{1+a^2/l^2}\left(d\tilde{\phi}+\Omega d\tilde{x}^4\right)\right)^2 + \frac{\cosh^2\theta}{1+a^2/l^2}(d\tilde{\phi}+\Omega d\tilde{x}^4)^2 - \frac{d\theta^2}{1-(a^2/l^2)\sinh^2\theta}.$$

[11] The differentials dx^4, $d\phi$ are still well defined and it is still acceptable to write the metric in terms of them.

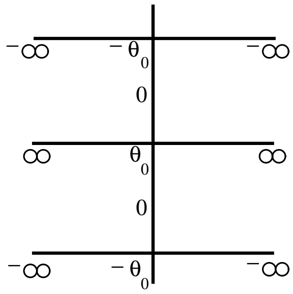

Figure 6: A skeleton diagram for the θ coordinate of the Kerr-AdS$_4$ $\pi/2$-bubble. Horizontal segments are where θ is spacelike and vertical where θ is timelike. Here, four-segment intersections are spinning $\tilde{x}^4\tilde{\phi}$ horizons.

On the other hand, we can form the double Killing bubble from (5.1). We take $x^4 \to ix^4$ and $\phi \to i\phi$ and then the metric becomes

$$ds^2 = \rho^2\left(\frac{dr^2}{\Delta} + \frac{d\theta^2}{1-\frac{a^2}{l^2}\cos^2\theta}\right) + \frac{\Delta}{\rho^2}\left(dx^4 - \frac{a\sin^2\theta}{1-a^2/l^2}d\phi\right)^2 - \\ - \frac{\sin^2\theta\left(1-(a^2/l^2)\cos^2\theta\right)}{\rho^2}\left(adx^4 - \frac{r^2+a^2}{1-a^2/l^2}d\phi\right)^2, \qquad (5.5)$$

where $\Delta = (r^2+a^2)(1+r^2/l^2) - 2Mr$, $\rho^2 = r^2 + a^2\cos^2\theta$.

We must twist like before: $\tilde{\phi} = \phi - \Omega x^4$, $\tilde{x}^4 = x^4$, $\Omega = a(1-a^2/l^2)/(r_2^2+a^2)$, $\tilde{x}^4 \simeq \tilde{x}^4 + \beta$ for some β, $\tilde{\phi}$ noncompact. So replace $d\phi \to d\tilde{\phi} + \Omega d\tilde{x}^4$ and $dx^4 \to d\tilde{x}^4$ in (5.5).

Take the case $l^2 > a^2$. Then $0 \le \theta \le \pi$ is fine, and we can continue $\theta \to i\theta$:

$$ds^2 = \rho^2\left(\frac{dr^2}{\Delta} - \frac{d\theta^2}{1-\frac{a^2}{l^2}\cosh^2\theta}\right) + \frac{\Delta}{\rho^2}\left(d\tilde{x}^4 + \frac{a\sinh^2\theta}{1-a^2/l^2}(d\tilde{\phi}+\Omega d\tilde{x}^4)\right)^2 + \\ + \frac{\sinh^2\theta(1-(a^2/l^2)\cosh^2\theta)}{\rho^2}\left(ad\tilde{x}^4 - \frac{r^2-a^2}{1-a^2/l^2}(d\tilde{\phi}+\Omega d\tilde{x}^4)\right)^2, \qquad (5.6)$$

where $\Delta = (r^2+a^2)(1+r^2/l^2) - 2Mr$, $\rho^2 = r^2 + a^2\cosh^2\theta$. With $\cosh^2\theta_0 = l^2/a^2$, $\theta = \theta_0$ is a horizon as before, and beyond it we have stationary regions of $\theta > \theta_0$. The skeleton diagram is shown in figure 7*a*. It is canonical to identify every other $0 \le \theta \le \pi$ horizontal segment. If on the other hand $a^2 > l^2$, then with $\cos^2\theta = l^2/a^2$, we imagine expanding out from $\theta = \pi/2$, we have θ_0 occurring before $\theta = 0, \pi$ and the skeleton diagram is shown in figure 7*b*. These solutions are nonsingular.

For $D \ge 5$, we must add $r^2\cos^2\theta d\Omega_{D-4}^2$ to the metric as in (5.1) [35] and also set $\Delta = (r^2+a^2)(1+r^2/l^2) - 2M/r^{D-5}$. The motivation is

$$d\Omega_{D-2}^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\Omega_{D-4}^2\,,$$

where $0 \le \theta \le \pi/2$. To get the $\pi/2$-bubble, send $\theta \to \pi/2 + i\theta$, $x^D \to ix^D$, $a \to ia$, and $d\Omega_{D-4}^2 \to -d\mathbf{H}_{D-4}^2$. Our motivating element becomes

$$d\mathrm{dS}_{D-2}^2 = -d\theta^2 + \cosh^2\theta d\phi^2 + \sinh^2\theta d\mathbf{H}_{D-4}^2\,.$$

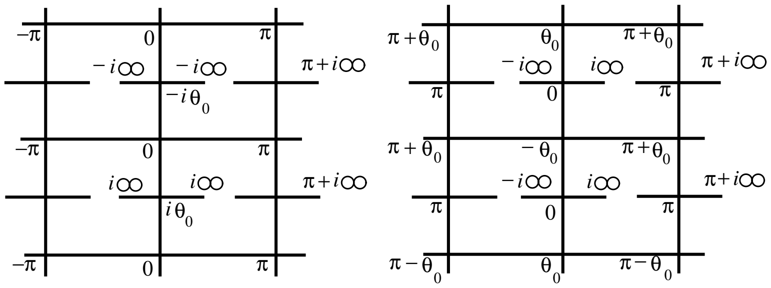

Figure 7: Skeleton diagram for the θ coordinate of the Kerr-AdS$_4$ double Killing bubble. Horizontal segments are where θ is spacelike and vertical where θ is timelike. (a) The figure on the left is the case where $l^2 > a^2$. (b) The figure on the right is the case where $l^2 < a^2$.

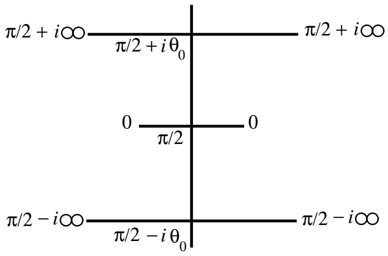

Figure 8: A skeleton diagram for the θ coordinate of the Kerr-AdS$_D$ $\pi/2$-bubble for $D \geq 5$, one turned on. At $\theta = 0$, a Killing direction closes the spacetime. (For $D \geq 6$, there is only one dS$_{D-4}$ leg and we have a $\vdash$ junction instead of $+$ junction at $\theta = \pi/2$.) At $\theta = \pi/2$, dS$_{D-4}$ becomes null and becomes $\mathbf{H}_{D-4}$. At $\theta = \pi/2 \pm i\theta_0$, there is a spinning horizon orbifold — this solution is singular.

Alternatively, we could have done $d\Omega^2_{D-4} \to d\mathrm{dS}^2_{D-4}$, $x^D \to ix^D$, $a \to ia$, where our motivating element becomes

$$d\mathrm{dS}^2_{D-2} = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\mathrm{dS}^2_{D-4}\,. \tag{5.7}$$

These two representations of dS$_{D-2}$ are connected together where either hyperbolic space or dS$_{D-4}$ degenerates to the null cone. A skeleton diagram is drawn in figure 8. This spacetime, however, is problematic — we have a compactification at $\theta = 0$ of (5.7) and another compactification condition at $r = r_2$. Generically, both Killing directions are compact, and the horizon at $\theta = \pi/2 \pm i\theta_0$ is thus an orbifold.

The double-Killing bubble, gotten from $x^D \to ix^D$, $a \to ia$, is nonsingular. There are two cases, $l^2 > a^2$ and $a^2 > l^2$. The skeleton diagrams are different from the $D = 4$ case and are shown in figures 9(a,b).

5.2 S-branes and anti-bubbles

The $\pi/2$-S-Kerr-AdS with one on can be motivated from the continuation and sign flip

$$\begin{aligned} d\Omega^2_{D-2} &= d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\Omega^2_{D-4} \\ \to d\mathbf{H}^2_{D-2} &= d\theta^2 + \cosh^2\theta d\phi^2 + \sinh^2\theta d\Omega^2_{D-4}\,. \end{aligned}$$

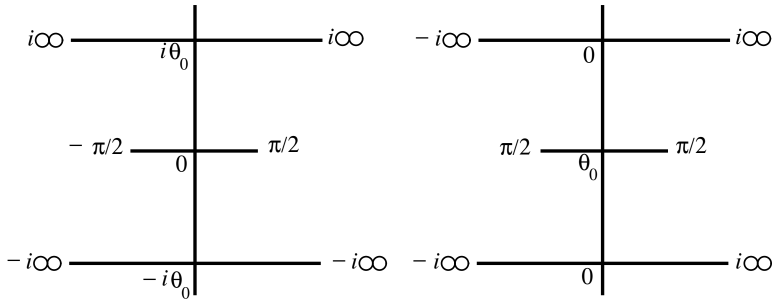

Figure 9: Skeleton diagrams for the θ coordinate of the Kerr-AdS$_D$ double-Killing bubble for $D \geq 5$, one turned on. At $\theta = \pm\pi/2$, the S^{D-4} closes the spacetime. The four-leg junctions are all spinning horizons. (a) The diagram on the left is for $l^2 > a^2$. (b) The diagram on the right is for $l^2 < a^2$.

The full continuation of Kerr-AdS is $\theta \to \pi/2 + i\theta$, $g_{\mu\nu} \to -g_{\mu\nu}$, $\phi \to i\phi$, $a \to ia$. The result has $\Lambda > 0$ and is

$$ds^2 = \rho^2 \left(-\frac{dr^2}{\Delta} + \frac{d\theta^2}{1-(a^2/l^2)\sinh^2\theta}\right) + \frac{\Delta}{\rho^2}\left(dx^D + \frac{a\cosh^2\theta}{1+a^2/l^2}d\phi\right)^2 + \\ + \frac{\cosh^2\theta}{\rho^2}(1-(a^2/l^2)\sinh^2\theta)\left(a dx^D - \frac{r^2-a^2}{1+a^2/l^2}d\phi\right)^2 + r^2\sinh^2\theta d\Omega^2_{D-4}\,, \quad (5.8)$$

where $\rho^2 = r^2 + a^2\sinh^2\theta$ and $\Delta = (r^2-a^2)(1+r^2/l^2) - 2M/r^{D-5}$. This has $W = 0$ at $\theta = \pm\theta_0$ where $\sinh\theta_0 = l/|a|$; θ is spacelike and for $r > r_2$, the Killing directions are $++$, so this closes the spacetime. The conditions at $\pm\theta_0$ are identical hence compatible and one Killing direction is noncompact to give a horizon at $r = r_2$. This solution is nonsingular.

The $\pi/2$-S-Kerr-dS solution has no $W = 0$ and has already been discussed, but we use $\pi/2$-S-Kerr-dS to construct anti-bubbles. The $\pi/2$-S-Kerr-dS$_D$ solution with one on is

$$ds^2 = \rho^2 \left(-\frac{dr^2}{\Delta} + \frac{d\theta^2}{1+(a^2/l^2)\sinh^2\theta}\right) + \frac{\Delta}{\rho^2}\left(dx^D + \frac{a\cosh^2\theta}{1-a^2/l^2}d\phi\right)^2 + \\ + \frac{\cosh^2\theta}{\rho^2}\left(1+\left(\frac{a^2}{l^2}\right)\sinh^2\theta\right)\left(a dx^D - \frac{r^2-a^2}{1-a^2/l^2}d\phi\right)^2 + r^2\sinh^2\theta d\Omega^2_{D-4}\,, \quad (5.9)$$

where $\rho^2 = r^2 + a^2\sinh^2\theta$ and $\Delta = (r^2-a^2)(1-r^2/l^2) - 2M/r^{D-5}$.

One anti-bubble is gotten from $x^D \to ix^D$, $\phi \to i\phi$, with motivating element

$$d\mathrm{AdS}^2_{D-2} = d\theta^2 - \cosh^2\theta d\phi^2 + \sinh^2\theta d\Omega^2_{D-4}\,.$$

This solution does not have $W = 0$ so it was already covered in the last section.

For $D \geq 5$, another anti-bubble is gotten from $x^D \to ix^D$, $a \to ia$, $d\Omega^2_{D-4} \to d\mathrm{dS}^2_{D-4}$. Since $W \propto 1 - (a^2/l^2)\sinh^2\theta$, the space closes at $\theta = \pm\theta_0$. But the space also closes at $r = r_2$ and these two conditions are not compatible, orbifolding the horizon that occurs at $\theta = \pm i\pi/2$.

We now investigate the usual S-Kerr-AdS and Kerr-dS anti-bubbles instead of the $\pi/2$-versions and find that generically they are all problematic, though there may be lower-dimensional parametric families or special cases that work.

The usual S-brane is gotten from the black hole by $g_{\mu\nu} \to -g_{\mu\nu}$, $\theta \to i\theta$, $d\Omega^2_{D-4} \to -d\mathbf{H}^2_{D-4}$. S-Kerr-AdS$_D$ with one on is

$$\begin{aligned} ds^2 &= \rho^2 \left(-\frac{dr^2}{\Delta} + \frac{d\theta^2}{1-(a^2/l^2)\cosh^2\theta} \right) + \frac{\Delta}{\rho^2} \left(dx^4 + \frac{a\sinh^2\theta}{1-a^2/l^2} d\phi \right)^2 + \\ &+ \frac{\sinh^2\theta(1-(a^2/l^2)\cosh^2\theta)}{\rho^2} \left(adx^4 - \frac{r^2+a^2}{1-a^2/l^2} d\phi \right)^2 + r^2\cosh^2\theta d\mathbf{H}^2_{D-4}\,, \end{aligned} \tag{5.10}$$

where $\rho^2 = r^2 + a^2\cosh^2\theta$ and $\Delta = (r^2+a^2)(1+r^2/l^2) - 2M/r^{D-5}$. Here $W = 0$ occurs, and this solution is problematic. Assuming $l^2 > a^2$ to get the right signature, the $\theta = 0$ and $\theta = \theta_0$ conditions are incompatible, forcing x^D to be compact and the $r = r_2$ horizon to be an orbifold.

S-Kerr-dS$_D$ is

$$\begin{aligned} ds^2 &= \rho^2 \left(-\frac{dr^2}{\Delta} + \frac{d\theta^2}{1+(a^2/l^2)\cosh^2\theta} \right) + \frac{\Delta}{\rho^2} \left(dx^4 + \frac{a\sinh^2\theta}{1+a^2/l^2} d\phi \right)^2 + \\ &+ \frac{\sinh^2\theta(1+(a^2/l^2)\cosh^2\theta)}{\rho^2} \left(adx^4 - \frac{r^2+a^2}{1+a^2/l^2} d\phi \right)^2 + r^2\cosh^2\theta d\mathbf{H}^2_{D-4}\,, \end{aligned} \tag{5.11}$$

where $\rho^2 = r^2 + a^2\cosh^2\theta$ and $\Delta = (r^2+a^2)(1-r^2/l^2) - 2M/r^{D-5}$.

The double Killing anti-bubble is gotten from (5.11) by $x^D \to ix^D$, $\phi \to i\phi$. The solution as written is then good down to $\theta = 0$ where we have a spinning Rindler horizon; then move up to $\theta = \pm i\pi/2$ where $\mathbf{H}_{D-4}$ becomes dS$_{D-4}$ and then to $\theta = \pm i\pi/2 \pm \theta_0$ with $\sinh\theta_0 = l/|a|$, where the space closes. The space closing here is generally incompatible with the $r = r_2$ condition, making the $\theta = 0$ horizon into an orbifold. The exception is $D = 4$ where there is no $\mathbf{H}_{D-4}$; this has no $W = 0$ and has already been discussed.

On the other hand, for $D \geq 5$, making an anti-bubble from $x^D \to ix^D$, $a \to ia$, $d\mathbf{H}^2_{D-4} \to d\mathrm{AdS}^2_{D-4}$ gives $W \propto 1-(a^2/l^2)\cosh^2\theta$. Assuming $l^2 > a^2$, the spacetime closes at $\theta = 0$ as well as $\theta = \theta_0$ and in general this is not compatible with the $r = r_2$ condition. Also there may be a 'ring' singularity $\rho^2 = 0$, although it does not propagate to large r.

6. Conclusions and relation to holography

In this paper we presented a procedure to generate time-dependent (and other black and anti-bubble) backgrounds starting from black holes solutions in (A)dS spacetime. We hope that our unified treatment of S-branes, bubbles and anti-bubbles with an emphasis on which solutions are possible, which are distinct, and what is their global structure including horizons and singularities, is useful to the reader. Some solutions in this paper are already known; several have been reexamined, reinterpreted or renamed (the 'anti-bubble') and several new solutions have been presented.

We have emphasized $D = 4,5$ $r\theta$-diagrams and θ-skeleton diagrams to keep track of spacetime regions and for pure (A)dS$_D$ for $D = 4,5$ we present various card diagrams in appendix. Our analytic continuation has been simple, involving only Killing directions and cosine directions. For $D = 4,5$ analytic continuation has been restated in terms of the card diagram technique of the γ-flip.

We find six types of spacetimes with a characteristic expected conformal boundary geometry. Black holes in AdS have $S^{D-2} \times \mathbb{R}_{\text{time}}$, bubbles have $\text{dS}_{D-2} \times S^1$, anti-bubbles have $\text{AdS}_{D-2} \times S^1$ and S-branes with $\Lambda < 0$ have $\mathbf{H}_{D-2} \times \mathbb{R}_{\text{time}}$. Black holes in dS have conformal boundary geometry $S^{D-2} \times \mathbb{R}_{\text{space}}$ and S-branes with $\Lambda > 0$ have $\mathbf{H}_{D-2} \times \mathbb{R}_{\text{space}}$. Solutions from Kerr-(A)dS which have $W = 0$ horizons, if they are good spacetimes, have a more complicated global structure for themselves and for their conformal boundaries.

Since many of the presented solutions are locally asymptotically (A)dS, it would be interesting to study them in the context of gauge/gravity dualities — the holographic results concerning some of the new spacetimes are forthcoming. The main tool that we use is the counterterm method proposed by Balasubramanian and Kraus in [37]. That is, to regularize the boundary stress tensor and the gravity action by supplementing the quasilocal formalism [38] with counterterms depending of the *intrinsic* boundary geometry. This way, the infrared divergencies of quantum gravity in the bulk are equivalent to ultraviolet divergences of dual theory living on the boundary. This method was also generalized to locally asymptotically dS spacetimes [39, 40]. However, unlike the AdS/CFT correspondence, the conjectured dS/CFT correspondence is far from being understood (see, e.g. [41] for a nice review).

Recently, Ross and Titchener [42] used the counterterm method to show that the AdS/CFT may teach us how to choose the right vacuum for the strongly-coupled CFT living on a dS background. The main idea is to use the black hole-AdS-bubble solution as a laboratory for studying the description of vacuum ambiguities in AdS/CFT. Also, Balasubramanian et al. [43] investigate the semiclassical decay of a class of orbifolds of AdS space via a bubble of nothing.

Using similar 'holographic' reasoning[12] to investigate some of the solutions presented in this paper, we hope to shed light on different aspects of the gauge/gravity correspondence for time-dependent backgrounds.

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A. Generalized card diagrams for $(\text{A})\text{dS}_4$, $(\text{A})\text{dS}_5$

Some of the solutions in this paper were found in analogy with card diagram techniques [17] (see also [51 – 53, 12, 54]). Furthermore the asymptotia of these solutions can be understood from the card diagram perspective. It is thus appropriate to give a small application of card diagrams to (anti-)de Sitter space in dimensions 4 and 5, where they have the requisite

[12] Other interesting examples of time-dependent AdS/CFT and dS/CFT correspondences can be found in [26, 34, 44 – 49]. It is also worth mentioning that, in a different context [50], some unexpected results were obtained for asymptotically AdS Taub-NUT spacetimes.

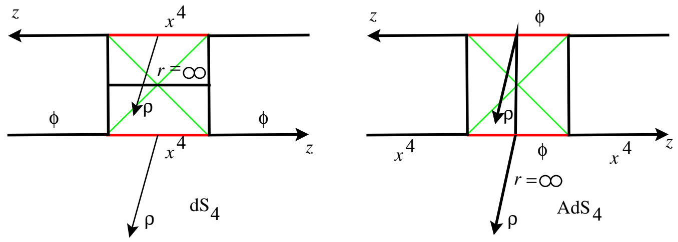

Figure 10: (a) On the left, we have dS_4 fibered by S^2. Turning the vertical square card on its side, we get (b) the diagram on the right; it is AdS_4 fibered by patched AdS_2. This is the same fibering as the $RNdS_4$ anti-bubble.

2 and 3 commuting Killing fields. The Weyl technique for Einstein's dynamical equation fails with a nonzero Λ. Nonetheless these spacetimes still have satisfying card diagrams. Here, we will not give a theory of generalized card diagrams, but rather just some examples which we can obtain by formal analogy to the 4d Reissner-Nordstrøm black hole. More conventional Penrose diagrams for (A)dS may be found in [55, 56].

The massless RN black hole of imaginary charge (to make it subextremal) has line element $\propto \frac{dr^2}{r^2-Q^2}+d\theta^2$. Once the non-Killing directions are of this form, we can immediately go to spherical prolate coordinates via $r = Q\cosh\zeta$; then $ds^2 \propto d\zeta^2 + d\theta^2$; and then to card diagram coordinates via $\rho = Q\sinh\zeta\sin\theta$, $z = Q\cosh\zeta\cos\theta$.

De Sitter 4-space has $\frac{dr^2}{1-r^2/l^2} + d\theta^2$. Set $u = 1/r$. Then we get $\propto \frac{du^2}{u^2-l^2} + d\theta^2$ and can proceed as above. The result is an elliptic card diagram with a rod horizon $-1/l < z < 1/l$, and the vertical square card above it is bisected halfway up at $u = 0$ (see figure 10*a*). Please note that for simplicity we have only drawn two cards at each 4-card horizon; see [17].

Turning the vertical card on its side via the γ-flip, we achieve AdS_4 in a coordinate system similar to the $RNdS_4$ anti-bubble solution (see figure 10*b*). An infinite stack of cards give periodic time. The $RNdS_4$ anti-bubble asymptotes to all the $r = \infty$ asymptotia drawn here. Figure 10*b* can be double Killing continued to give a card diagram suitable for understanding S-$RNdS_4$; this will be skipped for brevity.

To get AdS_4 fibered with spheres, we start with $\frac{dr^2}{1+r^2/l^2} + d\theta^2$. Let $u = 1/r$ as for de Sitter, and we get $\propto \frac{du^2}{u^2+l^2} + d\theta^2$. Now, the solution is superextremal and on a branched horizontal card. To go to spherical prolate coordinates, let $u = l\sinh\zeta$. The resulting card diagram is shown in figure 11. This card diagram can be double Killing continued to give AdS_4 fibered by dS_2, like the $RNAdS_4$ bubble; this will be skipped for brevity.

We give one more example of an interesting 4d de Sitter card diagram: the purely time-dependent one where dS_4 is fibered by azimuthal dS_2 and $\mathbf{H}_2$; see figure 12. Each 45-45-90 triangle with a vertical hypotenuse is a compactified representation of a half-plane vertical card without special null lines. The cards are compactified in precisely the same way as for a Penrose diagram. The S-$RNAdS_4$ solution asymptotes down through the $\mathbf{H}_2$-fibered region to the two $r = \infty$ regions drawn, with the exception of the 'point' on the right side (actually a ϕ-circle) where the x^4-circle would vanish.

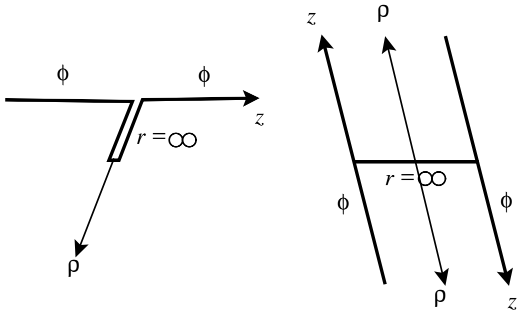

Figure 11: AdS_4 fibered by S^2. Shown as a doubly covered half-plane and then in a conformally fixed picture.

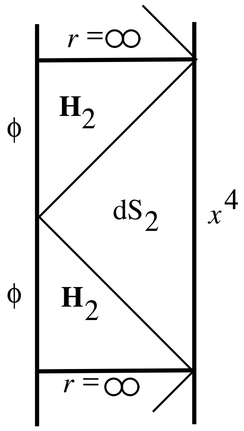

Figure 12: dS_4 fibered by azimuthal $\mathbf{H}_2$, dS_2. These vertical cards have been compactified.

Lastly we look at 5d case and find a card diagram which has both 5d de Sitter and anti-de Sitter in the same diagram. Take dS_5 fibered by S^3 with $d\Omega_3^2 = d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\psi^2$; it has $\frac{dr^2}{1-r^2/l^2} + r^2 d\theta^2$ just like the 4d case. We will get an elliptic card diagram. We want the rightward z-ray boundary to be $\theta = 0$ and now we want the leftward z-ray boundary to be $\theta = \pi/2$, not π. So in analogy with 4d RN, we need the metric to look like $\frac{dv^2}{v^2-v_0^2} + 4d\theta^2$.

To this end, let $u = 1/r^2$ and then $u = v + 1/2l^2$. The metric is $\propto \frac{dv^2}{v^2-1/4l^4} + 4d\theta^2$ and we let $v = (1/2l^2)\cosh\zeta$. The card diagram is then as follows: take the card diagram structure of the Schwarzschild black hole [17]; call the positive-mass external universe the 'primary' horizontal card and the negative-mass universe the 'secondary' horizontal card. Alternatively, take figure 10 and label the alternating levels of horizontal cards primary and secondary. Two primary horizontal cards and two vertical cards which connect at an $r = l$ horizon form a dS_5 of signature $++++-$. Each secondary horizontal card forms an AdS_5 of opposite signature $+----$. The horizontal edges of the vertical cards which are not horizons give $r = \infty$ regions for the dS_5 and AdS_5 universes that 'meet' there. Note that both $++++-$ dS_5 and $----+$ AdS_5 satisfy the Einstein-Λ equation with the same $\Lambda > 0$, and hence can appear on the same card diagram.

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These diagrams also apply to orbifolds of pure (A)dS space, such as the constant-curvature black hole of [57], which has a non-Killing horizon, and bears certain resemblance to a BHAdS bubble.

A more complete treatment of generalized card diagrams may appear elsewhere.

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